Int. J. Multiphase Flow, Vol. 2, pp. 471–475. Pergamon/Elsevier, 1976. Printed in Great Britain.

DROPLET SIZE RESULTING FROM BREAKUP OF LIQUID AT GAS–LIQUID INTERFACES OF LIQUID-SUBMERGED SUBSONIC AND SONIC GAS JETS

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(Received 15 June 1975)

1. INTRODUCTION

It has been demonstrated both analytically and experimentally (Chawla 1975a; Bell, Boyce & Collier 1972) that, due to existence of the Kelvin–Helmholtz instability at the gas–liquid interfaces of liquid-submerged subsonic and sonic gas jets, liquid at the interface breaks up and becomes entrained in the form of droplets in these gas jets. The study of this phenomenon has numerous industrial applications. For example, in the study of the fuel-failure propagation potential of fission-gas jet impingement in liquid metal cooled fast breeder reactor subassemblies, the liquid entrainment rate and droplet size control the rate of heat transfer in the impingement area of the fission-gas jet (Chawla 1975b). The study of the rate of entrainment and droplet size at the gas–liquid interface of a sonic gas jet submerged in a liquid is also of interest in the field of boiling water reactor safety (Chawla 1975b) and in a number of chemical-engineering processes.

An analysis of the rate of liquid entrainment at the gas–liquid interface of a liquid-submerged sonic gas jet has been presented previously (Chawla 1975b). The purpose of the present note is to obtain the droplet sizes due to breakup of the liquid at the interfaces of subsonic and sonic gas jets submerged in a liquid. The existing correlations (e.g. Hinze 1948, 1955) for droplet size do not explicitly include the effect of Mach number or compressibility of gas stream; the present analysis, however, explicitly obtains this dependence for both subsonic and sonic gas jets.

2. ANALYSIS

The mechanism of the Kelvin–Helmholtz instability, which is governed by transfer of energy to the liquid layer at the interface through the action of a pressure perturbation in the gas phase against the stabilizing forces due to surface tension and viscosity of liquid (Chawla 1975a), causes the disturbance at the interface to grow with time; when the amplitude of the disturbance becomes large enough, the liquid at the wave crests (protrusions into gas jets) is torn off by a gas jet. The size of the resulting droplets thus formed is obtained by utilizing Taylor's postulate (Taylor 1963) which states that

$$D \propto \lambda_m, \quad [1]$$

where λ_m is the wavelength of the disturbance at maximum instability. A detailed analysis of Kelvin–Helmholtz instability for a sonic gas jet is given by Chawla (1975a) and for a subsonic gas jet by Chawla (1974). For the present application, we give below approximate expressions for λ_m: for low-viscosity liquids and small-wave velocity of the disturbance

$$\lambda_m = \frac{3\pi\sigma(1-M^2)^{1/2}}{\rho_g U_g^2} \quad \text{for} \quad M < 1; \quad [2a]$$

$$\lambda_m^* = \left(\frac{2\pi}{0.803}\right)\left(\frac{\rho_g^*}{\rho}\right)^{1/5} \frac{\sigma}{\rho_g^* U_g^{*2}} \quad \text{for} \quad M = 1. \qquad [2b]$$

Here U_g, ρ_g, $M = U_g/U_g^*$ are, respectively, the gas jet velocity, density and Mach number at the throat conditions; U_g^*, ρ_g^* are, respectively, sonic velocity and critical density at throat conditions for the sonic gas jet, ρ, σ are, respectively, density and surface tension of the liquid. The corresponding conditions on the flow parameters of the gas–liquid system for which the above expressions are valid, are: for $M < 1$,

$$\left(\frac{\rho_g}{\rho}\right)^{1/2} \frac{\mu U_g}{\sigma(1 - M^2)^{1/4}} \ll 1; \quad \frac{(\rho_g/\rho)^{1/2}}{(1 - M^2)^{1/4}} \ll 1; \qquad [3a]$$

and for $M = 1$

$$\left(\frac{\rho_g^*}{\rho}\right)^{2/5}\left(\frac{\mu U_g^*}{\sigma}\right) \ll 1; \quad (\rho_g^*/\rho)^{2/5} \ll 1, \qquad [3b]$$

where μ is the dynamic viscosity of the liquid. The first of each of the above inequalities characterizes a low-viscosity liquid and the second characterizes a low wave velocity disturbance. A large number of gas–liquid systems of physical interest satisfy the above conditions. If, however, for a given system the above conditions are not satisfied, one should then obtain λ_m from a more exact analysis as given by Chawla (1974, 1975a).

The introduction of constant of proportionality in relationship [1] gives: for $M < 1$,

$$D = C_o \lambda_m \qquad [4a]$$

for low-viscosity liquids, the use of [2a] in the above equation gives

$$D = C_o \frac{3\pi\sigma(1 - M^2)^{1/2}}{\rho_g U_g^2} \qquad [4b]$$

or,

$$\frac{We}{(1 - M^2)^{1/2}} = 3\pi C_o; \qquad [4c]$$

for $M = 1$,

$$D = C\lambda_m^* \qquad [5a]$$

with the use of [2b] for low-viscosity liquids, the above equation becomes

$$D = C\left(\frac{2\pi}{0.803}\right)\left(\frac{\rho_g^*}{\rho}\right)^{1/5} \frac{\sigma}{\rho_g^* U_g^{*2}} \qquad [5b]$$

or,

$$\left(\frac{\rho}{\rho_g^*}\right)^{1/5} We = \frac{2\pi C}{0.803}, \qquad [5c]$$

where C and C_o are constants of proportionality and We is the Weber number. For $M \to 1$, [4a] and [5a] are combined to give the following composite expression:

$$D = C_o \lambda_m + C\lambda_m^*. \qquad [6]$$

This expression is valid for all Mach numbers up to and including unity. For example, at $M = 1$, the first term vanishes as can be seen from [2a] for low viscosity liquids and from figure 2 obtained from more exact analysis described in Chawla (1974) for liquids having finite viscosity (gas–liquid systems chosen for the latter demonstration are shown in the figure), and the remaining term corresponds exactly to [5a] for the sonic gas jet; we also note from figure 2 and [2a] that wavelength λ_m increases as Mach number decreases and thus the contribution of the second term in [6] becomes negligibly small as $M \to 0$ and [6] corresponds approximately to [4a]. The manner in which [6] is arrived at is consistent with the method proposed by Churchill & Usagi (1972) for obtaining a "composite" expression in terms of asymptotic solutions for large and small values of the independent variable.

3. CORRELATION WITH EXPERIMENTAL DATA

To obtain values for the empirical constants C_o and C, the experimental data of Gretzinger (1956) for sonic gas jets using both converging and impingement nozzles and of Weiss & Worsham (1959) for the subsonic gas jet were utilized. Figure 1 shows the plots of measured droplet diameter for the sonic gas jet as a function of wavelength, where wavelengths were obtained for the flow parameters covered in the experiment from a more exact analysis (Chawla

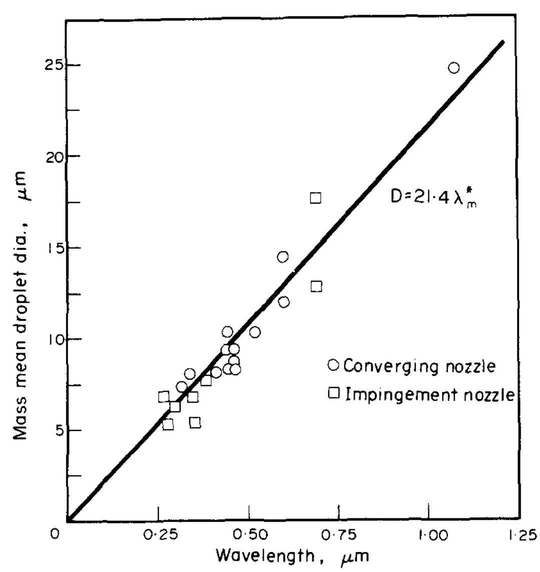

Figure 1. Correlation for the droplet size for a sonic gas jet using Gretzinger's data (1956).

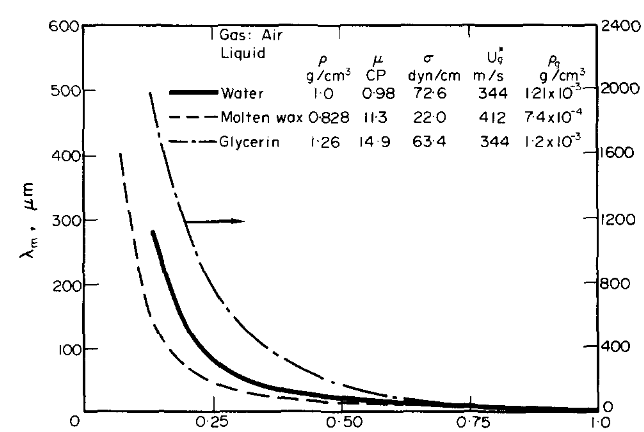

Figure 2. The variation of wavelength λ_m as a function of Mach number for a subsonic gas jet with liquids having finite viscosity.

1975) which allows for the effect of a finite viscosity of liquid and finite wave velocity of the disturbance. As can be seen, most of the data can be correlated satisfactorily on the basis of [6] (note that the term $C_o\lambda_m$ due to subsonic gas jet drops out at $M = 1$), yielding $C = 21.4$. Thus, comparison with the experimental data presented in figure 1 verifies the validity of [6] for Mach number equal to a value of unity. It now only remains to verify the validity of [6] for Mach number less than unity. For this purpose, we utilize Weiss & Worsham's (1959) experimental data obtained over the range of Mach numbers between 0.15 to 0.75 using air and molten wax as a gas–liquid system. Figure 3 shows the plot made on the basis of [6] of their data on droplet size utilizing values of wavelengths obtained from more exact analysis (Chawla 1974) corresponding to the range of flow parameters covered in their experiment; the values of λ_m^* correspond to sonic velocity at the throat temperature of the subsonic gas jet. As can be seen, the data are correlated satisfactorily with [6], yielding $C_o = 1.5$, and thus providing verification of [6] for Mach number

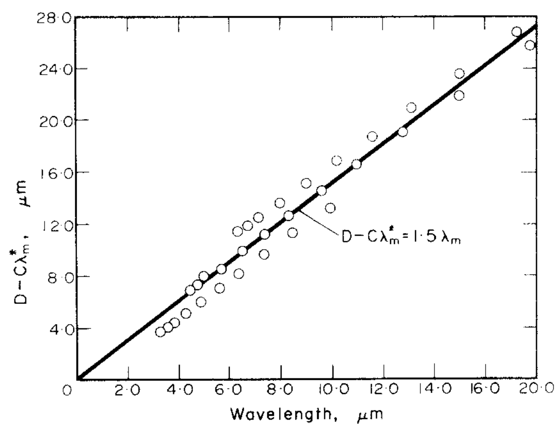

Figure 3. Correlation for the droplet size for a subsonic gas jet using the Weiss and Worsham data (1959).

less than unity. With this value of C_o, [4c] gives for $M \to 0$ a value of 14 for the Weber number. This value of the Weber number is in reasonably good agreement with the often quoted value of 13 (e.g. Hinze 1955) for nonviscous fluids.

Acknowledgements—The author would like to express his sincere thanks to A. Glassner for a careful editing of the manuscript and Debbie Lambert and Barbara Busch for an excellent job of typing the manuscript.

The work was performed under the auspices of the U.S. Energy Research and Development Administration.

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